Dyons in Nonabelian Born–Infeld Theory

A. Balaž[1], M. Burić[2] and V. Radovanović[3]

[1] *Institute of Physics, P. O. Box 57, 11001 Belgrade, Yugoslavia*

[2,3] *Faculty of Physics, University of Belgrade, P. O. Box 368, 11001 Belgrade, Yugoslavia*

Abstract

We analyze a nonabelian extension of Born–Infeld action for the $SU(2)$ group. In the class of spherically symmetric solutions we find that, besides the Gal'tsov–Kerner glueballs, only the analytic dyons have finite energy. The presented analytic and numerical investigation excludes the existence of pure magnetic monopoles of 't Hooft–Polyakov type.

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1 Introduction

Born-Infeld (BI) electrodynamics [1], was proposed in 1934. as a theory in which the energy of electrically charged point particle is finite, in contrast to the Maxwell electrodynamics. The Born-Infeld action is built similarly to the action of relativistic point particle and it introduces dimensional parameter, β, - the "maximal field strength". It is usually written in one of the following forms

$$S_{BI} = -\beta^2 \int d^4x \left(\sqrt{-\det\left(g_{\mu\nu} + \frac{1}{\beta}F_{\mu\nu}\right)} - \sqrt{-\det g_{\mu\nu}} \right) \tag{1}$$

$$= -\beta^2 \int d^4x \sqrt{-g} \left(\sqrt{1 + \frac{1}{2\beta^2}F_{\mu\nu}F^{\mu\nu} - \frac{1}{16\beta^4}\left(F_{\mu\nu}F^{*\mu\nu}\right)^2} - 1 \right) , \tag{2}$$

where $*$ is the Hodge-dual. This action has many interesting properties [2], among them duality symmetry, physical propagation, absence of birefringence, etc.

Actions of the BI-type arise in string/M theory in two main contexts. BI action represents the non-derivative part of the effective open string action. As it was shown in [3], the bosonic field partition function for the open string in an external field reduces to the BI lagrangian in the string theory limit.

[1]E-mail: antun@phy.bg.ac.yu
[2]E-mail: majab@ff.bg.ac.yu
[3]E-mail: rvoja@ff.bg.ac.yu

On the other hand, BI action is related to D-branes. This comes from the result that the effective action for the open strings ending on D-branes, after the integration of string degrees of freedom [4, 5], is Dirac-Born-Infeld (DBI) action:

$$S_{DBI} = -\int d^{p+1}x\sqrt{-\det(\eta_{\mu\nu} + F_{\mu\nu} + \partial_\mu y^i \partial_\nu y^i)}\ , \tag{3}$$

where $F_{\mu\nu}$ is the field strength and y^i's are scalar fields. BI action is obtained from (3) for $y^i = 0$. Conversely, DBI action can be related to BI action in higher dimensions by dimensional reduction.

The generalization of BI electrodynamics to nonabelian gauge theory is not unique. In the general case, if $F_{\mu\nu}$ is the field strength of the nonabelian gauge group $\mathcal{G}$ and $F_{\mu\nu} = F^a_{\mu\nu}T_a$ (T_a are the generators of $\mathcal{G}$, $[T_a, T_b] = if_{abc}T_c$), the "determinant" form of the action (1) is not equal to the "square-root" form (2). Different definitions of nonabelian Born-Infeld (NBI) lagrangians are possible, regarding the way of tracing the group indices. The symmetrized trace version of Tseytlin [6, 7, 8] is often regarded as the one which describes the non-derivative approximation of string theory; however, there are other proposals [9]. Usually NBI lagrangians cannot be put in the closed form in the component fields $F^a_{\mu\nu}$.

Following Gal'tsov and Kerner [10], in this paper we will analyze the simplest version of NBI action in which the trace over the group indices is done under the square-root sign. Gal'tsov and Kerner found particle-like finite energy solutions for the NBI action for the $SU(2)$ gauge group. Motivated by this result and by the fact that the dyonic solutions are of interest in the brane theory, we analyze a more general class of solutions. We also discuss the existence of pure monopole solutions.

2 Action and field equations

The initial point of our analysis is the following nonabelian Born-Infeld action in Minkowski space:

$$S = \frac{1}{4\pi}\int d^4x(1-\mathcal{R})\ , \tag{4}$$

where $\mathcal{R}$ is defined as

$$\mathcal{R} = \sqrt{1 + \frac{1}{2}F^a_{\mu\nu}F^{\mu\nu a} - \frac{1}{16}F^a_{\mu\nu}F^{*\mu\nu a}}\ . \tag{5}$$

We put that the maximal field strength equals unity, $\beta = 1$. Lorentz inices μ, ν run from 0 to 3 and we will often split them into the temporal part 0 and the spatial part, $i,\ j = 1,\ 2,\ 3$. The signature which we use is $(-,+,+,+)$. $F^a_{\mu\nu}$ are the field strengths of the $SU(2)$ gauge group,

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \epsilon^{abc} A^b_\mu A^c_\nu \ , \tag{6}$$

with $a,\ b = 1,\ 2,\ 3$. The equations of motion which follow from the NBI action (4) are

$$D_\mu P^{\mu\nu} = 0 \ , \tag{7}$$

where $P_{\mu\nu}$ are the "displacements" defined by

$$P^a_{\mu\nu} = \frac{\partial \mathcal{L}}{\partial F^{\mu\nu a}} = \frac{F^{\mu\nu a} - G F^{*\mu\nu a}}{\mathcal{R}} \quad . \tag{8}$$

The quantity F^* denotes the Hodge-dual of F

$$F^{*\mu\nu} = \frac{1}{2}\,\epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} \ , \tag{9}$$

and we use the shorthand notation

$$G = \frac{1}{4}\, F^a_{\mu\nu} F^{*\mu\nu a} \ . \tag{10}$$

The equations of motion (7) can be complemented with the Bianchi identities

$$D_\mu F^{*\mu\nu} = 0 \ . \tag{11}$$

It is important to note that NBI theory has the duality symmetry as BI:

$$F^{\mu\nu} \to P^{*\mu\nu} \ , \quad P^{\mu\nu} \to -F^{*\mu\nu} \ . \tag{12}$$

Duality invariance can be seen from the vacuum equations (7) and (11), too. It can be used to generate new vacuum solutions from the given ones.

The ansatz for the gauge potentials of [10] was the "monopole ansatz",

$$A^a_0 = 0 \ , \quad A^a_i = \epsilon_{\,aik} \frac{1 - w(r)}{r}\, \frac{x^k}{r} \ , \tag{13}$$

and it describes the purely magnetic configurations. Electric and magnetic fields are defined by:

$$E^a_i = F^a_{i0} \ , \quad B^a_i = \frac{1}{2}\,\epsilon_{ijk} F^a_{jk} \ . \tag{14}$$

We will generalize the ansatz (13) – in fact, we will consider the general spherically symmetric static potential of the $SU(2)$ group (Witten's ansatz,

[11]). It is given via three real functions $a_0(r)$, $a_1(r)$ and $w(r)$ of the radial coordinate r. The components of the gauge potential read:

$$A_0^a = a_0(r)\frac{x^a}{r} \ , \tag{15}$$

$$A_i^a = a_1(r)\frac{x^a x^i}{r^2} + \epsilon_{aik}\,\frac{1-w(r)}{r}\,\frac{x^k}{r} \ . \tag{16}$$

Here x^a, x^i and x^k $(a,\, i,\, k = 1,\, 2,\, 3)$ are the Cartesian coordinates. The field strengths for this ansatz are

$$E_i^a = a_0'\,\frac{x_i x_a}{r^2} - \frac{a_0 w}{r}\,\frac{x_i x_a - \delta_{ia} r^2}{r^2} \ , \tag{17}$$

$$B_i^a = -2\delta_{ia}\,\frac{1-w}{r^2} + \frac{(1-w)^2}{r^2}\,\frac{x_i x_a}{r^2} + \left(\frac{1-w}{r^2}\right)'\,\frac{x_i x_a - \delta_{ia} r^2}{r} + \frac{a_1 w}{r^2}\,\epsilon_{iak} x_k \tag{18}$$

and prime denotes the derivative $\frac{d}{dr}$. The square root $\mathcal{R}$ becomes

$$\mathcal{R} = \sqrt{1 + \frac{(1-w^2)^2}{r^4} + 2\frac{w'^2}{r^2} + 2\frac{a_1^2 w^2}{r^2} - 2\frac{a_0^2 w^2}{r^2} - {a_0'}^2 - \frac{[a_0(1-w^2)]'^2}{r^4}} \ . \tag{19}$$

In order to find the equations for $a_0(r)$, $a_1(r)$ and $w(r)$ we can consider the condition of extremality of the action or introduce the ansatz (16) into (7)–(11). After the integration of angular variables, the action is proportional to the lagrangian L,

$$L = \int_0^\infty r^2(\mathcal{R} - 1)dr \ . \tag{20}$$

Varying the unknown functions a_0, a_1 and w, we obtain the set of the equations:

$$w^2 a_1 = 0 \ , \tag{21}$$

$$(1-w^2)\left(\frac{[a_0(1-w^2)]'}{r^2\mathcal{R}}\right)' = \frac{2w^2 a_0}{\mathcal{R}} - \left(\frac{r^2 a_0'}{\mathcal{R}}\right)' , \tag{22}$$

$$w a_0\left(\frac{[a_0(1-w^2)]'}{r^2\mathcal{R}}\right)' = -\frac{2w(1-w^2)}{r^2\mathcal{R}} - \left(\frac{2w'}{\mathcal{R}}\right)' - \frac{w a_0^2}{\mathcal{R}} + \frac{w a_1^2}{\mathcal{R}} \ . \tag{23}$$

3 NBI dyons

The system of equations (21–23) is a complicated nonlinear system. We will search for particular solutions of this system with finite energy. The energy of the static configurations is equal to the negative value of the lagrangian,

$$M = \int_0^\infty r^2(1-\mathcal{R})dr \ . \tag{24}$$

The convergence of this integral on both boundaries imposes restrictions on the asymptotic behavior of the functions a_0, a_1 and w, which we will discuss later.

Let us first consider the simplest equation, (21): it implies that either $w(r) = 0$ or $a_1(r) = 0$. But one can see rather easily that the configuration $w(r) = 0$, $a_1(r) \neq 0$ is gauge equivalent to the configuration $w(r) = 0$, $a_1(r) = 0$. Indeed, for $w(r) = 0$ we obtain that (21) and (23) are identically fulfilled, leaving $a_1(r)$ undetermined. This means that $a_1(r)$ represents the gauge freedom. The value of $a_1(r)$ does not influence the values of the field strengths in the case $w(r) = 0$, as can be seen from (17–18). Therefore, we will always assume that $a_1(r) = 0$ and denote $a_0(r) = a(r)$ in the following, keeping the indexed notation like a_0, a_1, w_0 etc. for the coefficients in the power series expansions.

The solutions with $a(r) = 0$, $w(r) \neq 0$ were discussed by Gal'tsov and Kerner in detail. In this case, the equations of motion reduce to

$$\left(\frac{w'}{\mathcal{R}}\right)' = -\frac{w(1-w^2)}{r^2\mathcal{R}} \quad , \tag{25}$$

and the square root $\mathcal{R}$ to the expression

$$\mathcal{R} = \sqrt{1 + \frac{(1-w^2)^2}{r^4} + 2\frac{w'^2}{r^2}} \quad . \tag{26}$$

The simplest solution of (25), $w(r) = \pm 1$, is the pure gauge. $w(r) = 0$ is also a solution, and it has the form of the Dirac monopole: this is embedded $U(1)$ monopole. Its energy is finite:

$$M_e = \frac{\pi^{3/2}}{3\Gamma(3/4)^2} \approx 1.2360 \ . \tag{27}$$

There is also an infinite discrete set of finite energy solutions $w_n(r)$, $n \in \mathbb{N}$, the so-called Gal'tsov-Kerner glueballs. These solutions can be found numerically using the condition that function $w(r)$ with the allowed asymptotic forms at $r \to 0$ and $r \to \infty$ is smooth in the intermediate region. The asymptotic expansions are:

$$\begin{aligned} r \to 0 \quad &: \quad w(r) = 1 - br^2 + O(r^4) \ , \\ r \to \infty \quad &: \quad w(r) = \pm 1 + \frac{c}{r} + O\left(\frac{1}{r^2}\right) \ . \end{aligned} \tag{28}$$

Let us note that the solutions behaving at infinity as $w(r) \to 0$ are excluded, thus leaving only the configurations with no magnetic charge. Solutions $w_n(r)$ behave as magnetic dipoles and have energies which tend to the energy M_e of the embedded monopole as $n \to \infty$.

The other simple possibility, $w(r) = 0$, $a(r) \neq 0$, is also nontrivial. The equations of motion in this case reduce to

$$\left(\frac{a'}{r^2\mathcal{R}}\right)' = -\left(\frac{r^2 a'}{\mathcal{R}}\right)' , \tag{29}$$

where now we have

$$\mathcal{R} = \sqrt{\frac{(1+r^4)(1-a'^2)}{r^4}} \ . \tag{30}$$

This equation can be solved explicitly and its solution is a two-parameter family

$$a(r;\ C,\ \alpha) = C \pm \int_0^r \sqrt{\frac{\alpha-1}{\alpha+r^4}}\, dr \ , \tag{31}$$

where C and α are the integration constants and $\alpha > 1$. As the energy and the field strenghts do not depend on C and the equations are invariant under $a(r) \to -a(r)$, we will take $C = 0$ and the $+$ sign in front of the square root. The explicit form of the solution is given in terms of the elliptic integral [12],

$$a(r;\ \alpha) = \frac{1}{2}(\alpha-1)^{1/2}\alpha^{-1/4} F\left(\arccos\frac{\sqrt{\alpha}-r^2}{\sqrt{\alpha}+r^2}\ ,\ \frac{1}{2}\right) \ . \tag{32}$$

The function $a(r;\ \alpha)$ is shown in the Figure 1 for different values of α. The limiting value of the parameter, $\alpha = 1$, gives $a(r) = \text{const}$, a configuration which is gauge equivalent to the embedded monopole $w(r) = 0$, $a(r) = 0$. The energy of the solution (31) is

$$M(\alpha) = \frac{\pi^{3/2}}{\Gamma(3/4)^2}\,\frac{1}{2\alpha^{1/4}}\,\left(1-\frac{\alpha}{3}\right) \ . \tag{33}$$

It is unbounded below with the maximum M_e at $\alpha = 1$. We observe that the existence of the electric field decreases the total energy.

We call the solution (31) dyon [13], as in the asymptotic region, $r \to \infty$, the behavior of the electric and magnetic fields is given by

$$E_i^a \sim \sqrt{\alpha-1}\ \frac{x_i x_a}{r^4}\ ,\quad B_i^a \sim -\frac{x_i x_a}{r^4}\ , \tag{34}$$

and describes the field strenghts of point-like sources. The "electric charge" of the source is proportional to $\sqrt{\alpha-1}$, while the "magnetic charge" is 1.

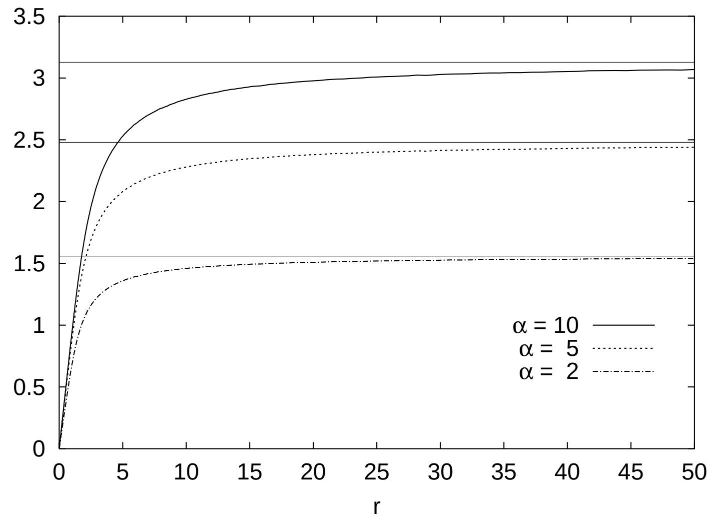

Figure 1: Dyon solution for various values of α.

Let us discuss the duals of the forementioned solutions. One defines the splitting of the displacement tensor in terms of the vectors D_i^a and H_i^a as

$$P_{i0}^a = D_i^a \ , \quad P_{ij}^a = \epsilon_{ijk} H_k^a \ . \tag{35}$$

The duality transformation (12) can then be reexpressed as

$$E_i^a \to -H_i^a = -\frac{B_i^a - GE_i^a}{\mathcal{R}} \ , \quad B_i^a \to D_i^a = \frac{E_i^a + GB_i^a}{\mathcal{R}} \ . \tag{36}$$

In the case of Gal'tsov-Kerner glueballs we have $E_i^a = 0$, $G = E_i^a B_i^a = 0$, so the duality transforms

$$E_i^a \to -\frac{B_i^a}{\mathcal{R}} \ , \quad B_i^a \to \frac{E_i^a}{\mathcal{R}} = 0 \ . \tag{37}$$

This means that from the magnetic dipole solution we obtain purely electric solution, which behaves as a dipole since $\mathcal{R} \to 1$ asymptotically.

In the case of a dyon we see that $G \sim r^{-4}$ and $\mathcal{R} \sim 1$ at infinity. The leading behavior of the transformed configuration will be

$$B_i^a \sim -\frac{x_i x_a}{r^4} \ , \quad E_i^a \sim \sqrt{\alpha - 1}\, \frac{x_i x_a}{r^4} \ , \tag{38}$$

i. e., the electric and magnetic charges interchange.

4 General case

We now turn to the analysis of the general case, $w(r) \neq 0$, $a(r) \neq 0$. The first condition that we want to impose on our solutions is finiteness of the energy. This condition restricts the possible behavior of the functions $w(r)$, $a(r)$ at the boundaries of the integral (24). If we expand $a(r)$ and $w(r)$ in the power series around $r = 0$, we conclude that (24) converges if there are no poles in the series, i. e. if they are of the form:

$$a(r) = \sum_0^\infty a_n r^n \ , \quad w(r) = \sum_0^\infty w_n r^n \ . \tag{39}$$

When we analyze the other boundary, $r \to \infty$, we obtain the similar asymptotics:

$$a(r) = \sum_0^\infty A_n r^{-n} \ , \quad w(r) = \sum_0^\infty W_n r^{-n} \ , \tag{40}$$

but now the convergence imposes $W_0 A_0 = 0$.

In order to analyze the relations among the coefficients in (39–40) further, we will assume that the equations of motion are satisfied order by order in r (or respectively in $1/r$).

Case $r \to 0$. From the equation (22) we obtain that w_0 must be ± 1 or 0. As the equations are invariant to the transformation $w(r) \to -w(r)$ and to $a(r) \to -a(r)$, we will discuss only $w_0 = 0$ and $w_0 = 1$. The similar situation will repeat in the next case.

For $w_0 = 0$ the equation (22) gives: $w_1 = w_2 = w_3 = \ldots = 0$; the function $w(r)$ vanishes. At the same time, from (21) we get $a_2 = a_3 = a_4 = 0$, $a_5 = \frac{a_1(a_1^2-1)}{10}$, $a_6 = a_7 = a_8 = 0$, $a_9 = \frac{a_1(a_1^2-1)^2}{24}$, etc. We also obtain that $|a_1| < 1$. Thus, both expansions show that this case corresponds to the dyon solution (31) with $a_1 = \sqrt{\frac{\alpha-1}{\alpha}}$.

For $w_0 = 1$ we get

$$\begin{aligned} a(r) &= a_1 r + a_3 r^3 + O(r^5) \ , \\ w(r) &= 1 + w_2 r^2 + w_4 r^4 + O(r^5) \ , \end{aligned} \tag{41}$$

where

$$a_3 = \frac{8a_1^3 w_2 + 8a_1 w_2^3 - 2a_1 w_2}{10a_1^2 - 20w_2^2 - 5} \ ,$$

$$w_4 = \frac{6w_2^2 + a_1^4(2 + 20w_2^2) + 16w_2^4(7 + 22w_2^2) - a_1^2(1 + 42w_2^2 + 408w_2^4)}{20(1 - a_1^2 + 4w_2^2)(1 - 2a_1^2 + 4w_2^2)} \ ,$$

etc., and $|a_1| < \frac{1}{\sqrt{3}}$. We will analyze this asymptotics in the following, let us just note here that for $a_1 = 0$ it is the one obtained in [10].

Case $r \to \infty$. We consider separately the possibilities $W_0 = 0$ and $A_0 = 0$.

If $W_0 = 0$, the assumption that the equations of motion are satisfied order by order in $1/r$ leads to $W_1 = W_2 = W_3 = \ldots = 0$. For coefficients of $a(r)$ we get $A_2 = A_3 = A_4 = 0$, $A_5 = -\frac{A_1(A_1^2+1)}{10}$, $A_6 = A_7 = A_8 = 0$, $A_9 = \frac{A_1(A_1^2+1)^2}{24}$, etc. Again, we obtain the power series expansion of the dyon (31), in this case around infinity.

For the second possibility, $A_0 = 0$, the solutions behave asymptotically as

$$\begin{aligned} a(r) &= \frac{A_2}{r^2} + \frac{A_3}{r^3} + \frac{A_4}{r^4} + O\left(\frac{1}{r^5}\right) \\ w(r) &= 1 + \frac{W_1}{r} + \frac{W_2}{r^2} + \frac{W_3}{r^3} + \frac{W_4}{r^4} + O\left(\frac{1}{r^5}\right) \ , \end{aligned} \tag{42}$$

where the following relations are fulfilled

$$A_3 = A_2 W_1 \ , \quad A_4 = \frac{18 A_2 W_1^2 - A_2^3}{20} \ , \quad W_2 = \frac{6 W_1^2 - A_2^2}{8} \ ,$$

$$W_3 = \frac{22 W_1^3 - 9 A_2^2 W_1}{40} \ , \quad W_4 = \frac{17 A_2^4 - 540 A_2^2 W_1^2 + 772 W_1^4}{1920} \ .$$

From this analysis we see that, in order to find new solutions, we need to join the asymptotics (41) and (42) smoothly. Our first attempt was to do the numerical integration from $r = 0$ to the right or from $r = \infty$ to the left, with the initial conditions defined appropriately. Doing this, we obtain the generic solution of a typical form shown in the Figure 2. The coupling of $a(r)$ and $w(r)$ induces the oscillations of $w(r)$ which reduce its initial value 1 for $r = 0$ to 0 for $r = \infty$. This solution is interesting, as it has the behavior of 't Hooft-Polyakov monopole. However, it is numerically unstable: if we keep the same values of w_2, a_1 and r_i but decrease the integration step h, we obtain the functions given in the Figure 3. The oscillations of $w(r)$ increase to a larger region of r, while the asymptotic value of $a(r)$ changes. We conclude that the solutions of this type are nonanalytic. Indeed, from the previous discussion of asymptotics we know that $w(r) \to 0$ as $r \to \infty$ is compatible only with $w(r) = 0$. Further numerical analysis of energy confirms this conclusion: the values of energy differ for orders of magnitude for different integration steps and therefore signal that the energy diverges. We see that in the NBI case, as in the pure Yang-Mills theory, $w(r) = 0$ and $w(r) = 1$ are separated by

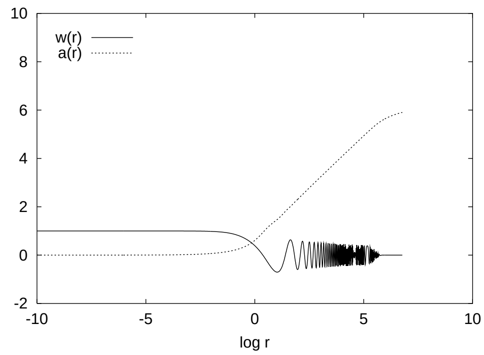

Figure 2: Solution for the parameters $w_2 = -10$, $a_1 = 0.5$ and integration step $h = 10^{-3}$. The initial point of integration is $r_i = 10^{-10}$.

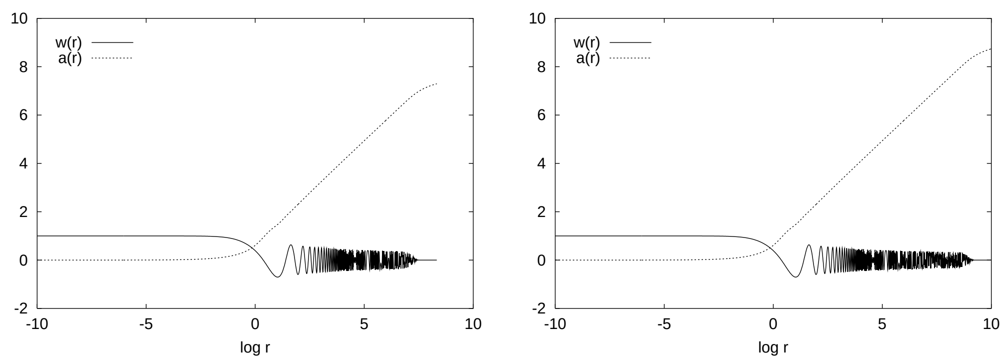

Figure 3: Solutions for parameters $w_2 = -10$, $a_1 = 0.5$, $r_i = 10^{-10}$ and integration steps $h = 10^{-4}$ (left) and $h = 10^{-5}$ (right).

an infinite energy barrier and it is impossible to find the solution of finite energy which interpolates between them.

The second possibility for numerical investigation is to start the integration from both sides $r = 0$ and $r = \infty$ with the given asymptotics, and try to join smoothly the solution in the intermediate region by varying the parameters w_2, a_1, W_1 and A_2. A numerical programme which handles this type of boundary conditions [14] was made, and proved to be correct and very efficient in the simple case of small a_1, A_2 (glueballs). However, no new solutions were found using this programme for a wide range of initial parameters. This might be a consequence of some weaknesses of the implemented variational procedure (Newton-Raphson), due to the high dimensionality of the parameter space. We are, however, inclined to interpret this as a strong numerical evidence that there are no further finite-energy solutions of the system (21–23).

5 Conclusions

The set of equations (21–23), which represent the equations of motion for the static spherically symmetric configurations of $SU(2)$ NBI action (4), is analyzed. The asymptotic analysis shows that, if one imposes finiteness of energy, there are only three possible types of solutions: glueballs, dyons and solutions of the form (39)–(41).

Dyon solutions are of importance in the brane-theory, as they represent strings ending on three-brane [5]. The name dyon, introduced after [13], is used in the generalized sense: there is no Higgs field to determine the unbroken $U(1)$ group. As in the case of Julia-Zee dyon, the electric charge of this solution is continuous while the magnetic charge is 1. However, the hope that the components A_0^a of the vector potential (given via the function $a(r)$) can, through the nonlinear interaction, take the role of Higgs and counterbalance the magnetic field to produce the monopole of the 't Hooft-Polyakov type failed. Instead of the exponential decay, $a(r)$ induces the oscillations of $w(r)$ with infinite energy. This could be expected from the fact that the change of the action from Yang-Mills to NBI does not change the topology of the fields which are included, necessary for the existence of monopole [15]. The solutions of the NBI models with Higgs fields were discussed in [16, 17].

Finally, let us add that, although the solutions of the third mentioned type are allowed by the energy considerations, we have a strong numerical indication that they do not exist. This problem might deserve further numerical analysis.

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References

[1] M. Born and L. Infeld, Proc. Roy. Soc. **A144** (1934) 425

[2] I. Bialynicki-Birula, Non-linear electrodynamics: Variations on a Theme of Born and Infeld, in *Quantum Theory of Fields and Particles*, eds. B. Jancewicz and J. Lukierski, World Scientific, Singapore, 1983.

[3] E. S. Fradkin and A. A. Tseytlin, Phys. Lett. B **163** (1985) 123, A. A. Tseytlin, *Born-Infeld action, supersymmetry and string theory*, preprint hep-th/9908105

[4] R. G. Leigh, Mod. Phys. Lett. **A4** (1989) 2767, J. Polchinski, Phys. Rev. Lett. **75** (1995) 4724, C. G. Callan and J. Maldacena, Nucl. Phys. B **513** (1998) 198

[5] G. W. Gibbons, *Aspects of Born-Infeld Theory and String/M Theory*, preprint hep-th/0106059

[6] A. A. Tseytlin, Nucl. Phys. B **501** (1997) 41

[7] D. Brecher, Phys. Lett. B **442** (1998) 117

[8] V. V. Dyadichev and D. V. Gal'tsov, Nucl. Phys. B **590** (2000) 504

[9] N. Grandi, E. F. Moreno and F. A. Schaposnik, Phys. Rev. D **59** (1999) 125014, J. H. Park, Phys. Lett. B **458** (1999) 471

[10] D. Gal'tsov and R. Kerner, Phys. Rev. Lett. **84** (2000) 5955

[11] M. S. Volkov and D. V. Gal'tsov, Phys. Rep. **319** (1999) 1

[12] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions*, Dover Publications, New York, 1972.

[13] B. Julia and A. Zee, Phys. Rev. D **11** (1975) 2227

[14] W. H. Press, B. P. Flannery, S. A. Teukolsky and W. T. Vetterling, *Numerical Recipes in C*, 2nd edition, Cambridge University Press, Cambridge 1992.

[15] G. 't Hooft, Nucl. Phys. B **79** (1974) 276

[16] N. Grandi, R. L. Pakman, F. A. Schaposnik and G. Silva, Phys. Rev. D **60** (1999) 125002

[17] Y. Brihaye and B. Hartmann, *The Born-Infeld Sphaleron*, preprint hep-th/0110053